Rocket Motor Performance Prediction and Simulations

This will be a somewhat simplified introduction to rocket motor performance prediction and simulation. The key to performance prediction and simulation is to have "good" numbers for propellant physical and ballistic characteristics. At a minimum, one must have a mathematical model for burn rate versus pressure, propellant density, a value for the specific heat ratio, a value for molecular weight of combustion products, a value for chamber temperature, and an algorithm for calculating thrust coefficient values at each pressure calculated. The AED (Netherlands) program GDPL seems to use this method. My model (the Rogers Aeroscience model seems to be similar) requires models or algorithms for burn area, burn rate, chamber pressure, and thrust coefficient, values for specific heat ratio, and propellant density. For black powder we actually need two models each for burn rate (as determined by chamber pressure) and for chamber pressure (as determined by Kn). This is because there are breaks in both curves at a pressure somewhere between 70 and 100 psia. When we first started to do some serious characterization work, but with cruder tools, we saw breaks close to 100 psia. Currently we are seeing breaks in curves in 75 to 100 psia range. I am not sure if this is real or if test and calculation methods contribute to this shifting of break points also.

Burn rate is mathematically defined by an equation of the form $r = aP_c^n$ where r is the burn rate in inches per second, a is the burn rate coefficient, and n is the burn rate exponent. Kn, the burn area ratio, is defined as burn area divided by nozzle throat area. It is also mathematically defined as $Kn = bP_c^{(1-n)}$ where Kn is burn area ratio, b is Kn coefficient, and (1-n) is Kn exponent. Our experimental values for (1-n) vary from calculated values using our experimental value for n, but act as a cross check on our calculations as they are normally close unless there are errors in our experiments or calculations. A little manipulation of our equations allow us to come up with an equation for chamber pressure as related to Kn. This is of the form $P_c = cKn^{(1/1-n)}$ where Pc is chamber pressure in psia, c is the pressure coefficient and (1/(1-n)) is the pressure coefficient.

Once we have the required starting info we can proceed with performance calculations. Our particular simulation method (pretty standard – sometimes referred to as "burn-back" analysis in some of the literature entails first calculating the burn area and Kn which, as we have said, is just burn area divided by nozzle throat area. We then plug in that value into our pressure calculation to calculate pressure. The pressure value is used to calculate the burn rate for the next step and to calculate the current thrust coefficient. One definition of thrust is $F = C_f P_c A_t$ where C_f is the thrust coefficient, P_c is chamber pressure (psia), and A_t is nozzle throat area (sq. in.). We use this definition to calculate thrust. We assume no change in throat dimensions in our model whether due to erosion or buildup. Some simulations include nozzle erosion. We use burn rate to calculate the distance burned in our next time increment (currently using increments of 0.002 seconds to be consistent with thrust stand program I use – QA's stand program uses 0.01 time steps to conserve space etc. – Still very accurate). Our next burn area is then calculated. This process is repeated until there is no propellant left to burn. One of our simplifying assumptions is that burning progresses in parallel layers. Another is that we model the tip of the drift as a hemisphere (much simpler to calculate with only a small loss in accuracy). Yet another simplification is to model the edge of the nozzle shape (where it contacts the propellant) as a 45-degree taper at the intersection with casing. We disregard any erosive burning effects or possible accelerated burning at edges of grain during last half of burn for long propellant burns that might be caused by flexure (or ballooning) of casing (possibly due to internal pressure being exerted on greater and greater surface area). These are all things that might be modeled if we had sufficient knowledge of how to do so. In spite of these simplifications, our method does give relatively realistic values and allows us to see the magnitude of changes we can expect with changes in powder lots etc. We also neglect such things as losses due to solids in the exhaust, heat losses, losses attributable to nozzle half angle etc. etc. Our methods of characterization of powder lots are also not perfect, but again give us information that is good enough for normal everyday use. More and better and more expensive test equipment would always be nice, but not necessarily cost effective. It's very easy to spend more money on information than it is worth.

We will briefly discuss the effects of burn rate exponent values. Black powder usually has a burn rate exponent around 0.4 to 0.5 at low pressures and around 0.1 at higher pressures. Lower values are nicer as they produce smaller, more controllable pressure excursions when changes in burn area occur (intentional or otherwise). A burn rate exponent of one or greater guarantees that an explosion will occur as there is then no way that gases can escape faster than they are produced. Since we have a higher

exponent at low pressures, problems while we are increasing to our peak thrust have a greater potential of accelerating beyond control than problems after reaching peak thrust. This may be the main reason I have never seen a catastrophic failure of motor after reaching peak thrust and starting the sustained thrust portion of operation.